ABSTRACT. A desired weld feature such as geometry can be produced using multiple sets of welding variables, i.e., different combinations of arc current, voltage, welding speed, and wire feed rate. At present, there is no systematic methodology that can determine, in a realistic time frame, these multiple paths based on scientific principles. Here we show that the various combinations of welding variables necessary to achieve a target gas metal arc (GMA) fillet weld geometry can be systematically and quickly computed by a real-number-based genetic algorithm and a neural network that has been trained with the results of a heat transfer and fluid flow model. The neural network is computationally efficient and, because of its origin, its input and the output obey the equations of conservation of mass, momentum, and energy. A genetic algorithm is used to determine a population of solutions by minimizing an objective function that represents the difference between the calculated and the desired values of the penetration, throat, and leg length. The model proposed here is different from traditional reverse models, since they cannot predict the welding variables needed to achieve a target weld geometry. The combinations of welding variables needed to achieve a target gas metal arc fillet weld geometry can be systematically and quickly computed by a real-number-based genetic algorithm and a neural network.

Introduction

In recent years, phenomenological models of various fusion welding processes such as gas tungsten arc (Refs. 1–10), gas metal arc (GMA) (Refs. 11–16), and laser beam welding (Refs. 17–19) have been developed to better understand physical processes in welding and calculate the weld geometry (Refs. 3–10), cooling rate (Refs. 4, 11–13), and other weld attributes such as weld metal phase composition (Refs. 4, 8), grain structure (Refs. 5, 6), and inclusion structure (Ref. 7). Although these powerful models have provided significant insight about the effect of various welding variables, their applications have been rather limited (Refs. 20–22) for several reasons. First, the models are comprehensive and require a significant amount of computer time. Second, they are designed to calculate temperature and velocity fields for a given set of welding variables, i.e., they are unidirectional in nature. In other words, they cannot predict the welding variables needed to achieve a target weld geometry.

What is very much needed, and not currently available, is for the models to have a capability to offer various choices of welding variable combinations, each capable of producing a target weld attribute. Traditional reverse models cannot produce multiple solutions and, in most instances, cannot confirm to any phenomenological laws.

Three main requirements need to be satisfied by a model for systematic tailoring of a weld attribute such as weld geometry, based on scientific principles. First, the model should be capable of capturing all the major complex physical processes occurring during GMA welding. Second, the model must have a bidirectional capability. In other words, in addition to the capability of the traditional unidirectional, forward models to compute the welding shape and size from a given set of welding variables, it should also have the inverse modeling ability, i.e., it should be able to systematically predict welding variables needed to produce a target weld geometry. Finally, the GMA welding system is highly complex and involves nonlinear interaction of several welding variables (Refs. 11–16, 24). As a result, a particular weld attribute such as the geometry can be obtained via multiple paths, i.e., through the use of various sets of welding variables. What is very much needed, and not currently available, is for the models to have a capability to offer various choices of welding variable combinations, each capable of producing a target weld attribute.

KEYWORDS

Neural Networks
Gas Metal Arc Welding
Fillet Welds
Genetic Algorithm
Heat Transfer Models
Fluid Flow Models
each iteration is modified to a different, more appropriate solution (Refs. 25, 26). Therefore, a combination of one of these classical optimization methods with the phenomenological model can provide only a single local optimum solution in situations where multiple solutions exist. In contrast, genetic algorithms (GA) mimic nature’s evolutionary principles to derive its search toward a population of optimal solutions (Refs. 25–28). In the context of welding, a GA can systematically search for multiple combinations of welding variable sets that comply with the phenomenological laws of welding physics and improve with iterations (Refs. 20–22).

Recently, Kumar and DebRoy (Ref. 20) and Mishra and DebRoy (Refs. 21, 22) developed bidirectional phenomenological models of GMA fillet welding and GTA butt joint welding, respectively, by coupling a genetic algorithm-based optimization method with three-dimensional heat transfer and fluid flow model. They showed that the above approach can predict multiple combinations of welding variables to achieve a target geometry. However, these models (Refs. 20–22) are unsuitable for practical applications, since they require several days of computer calculations. Kumar and DebRoy (Ref. 20) used a parallel computing facility, i.e., running their model on multiple processors simultaneously to reduce computational time. Since it is very hard to maintain such a sophisticated computing facility in a manufacturing industry, their model can only be used for research purposes. Unless a model can do calculations in a reasonable time, it is unlikely to find widespread practical applications.

In gas metal arc welding, the effect of welding variables on the weld geometry is nonlinear and highly complex. A well-trained and rigorously tested neural network (Refs. 29–31) can be used in place of a phenomenological model to capture the correlations between different welding variables and weld attributes. The neural network models are able to predict the outputs for different welding conditions rapidly (Refs. 29–31). With the improvements in computational hardware in recent years, a large volume of training and validation data can be generated with a well-tested numerical heat transfer and fluid flow model in a realistic time frame. A neural network trained with the results of a numerical heat transfer and fluid flow model can correlate various output variables such as the weld pool geometry, cooling rate, liquid velocities, and peak temperatures with all the major welding variables and material properties. Furthermore, such correlations satisfy the basic scientific phenomenological laws expressed in the equations of conservation of mass, momentum, and energy.

We show here that multiple sets of welding variables that are capable of producing a target weld geometry can be calculated in a realistic time frame by coupling a genetic algorithm with a neural network model of gas metal arc fillet welding that has been trained with the results of a well-tested heat transfer and fluid flow model.

Mathematical Model

The main computational engine used here is a neural network model (Ref. 29), which is trained and validated using the results of a well-tested heat transfer and fluid flow model (Refs. 11–16). The neural network model includes all the welding variables and material properties as input and provides weld dimensions, peak temperatures, maximum velocities, and the...
Objective function value: Value of objective function determines if a chromosome/individual survives or dies

Parents: Chromosomes/individuals participating for creating new individuals (or offsprings)
Parents: e.g., (1.10, 1.70, 1.56, 1.34), (1.23, 1.65, 1.75, 1.45)

Objective function value: Value of objective function calculated for each set of input variables using Equation 2.

Equation 3 is made nondimensional to preserve the importance of all four welding variables by making their nondimensional values comparable in magnitude. The GA produces new individuals, or sets of welding conditions, with iterations based on the evolutionary principles (Refs. 20–22, 26–28) as explained in the Appendix. Table 1 provides the explanation of various terminology used in GA related to welding.

Genetic algorithms work with a set of “individuals,” a population where each individual is a solution of a given problem. The initial population defines the possible solutions of the optimization problem, i.e., sets of welding variables that completely define a weld such as current, voltage, welding speed, contact tube-to-workpiece distance, and wire feed rate. There are two popular ways of representing the variables in the population in GA: binary and real numbers. Generally, binary representation of variables converges slowly compared to the real representations. In addition, since the binary genetic algorithm has its precision limited by the binary representation of variables, using real numbers allows representation to the machine precision. The real coded genetic algorithm also has the advantage of requiring less storage than the binary GA because a single floating point number represents a variable instead of many integers having values 0 and 1. The other important advantage of using real coded GA is its accuracy and precision in representing the variables in continuous search space.

The genetic algorithm (GA) used in the present study is a parent centric recombination (PCX) operator-based generalized generation gap (G3) model (Refs. 20–22, 27–29). The generic parent-centric recombination operator (PCX) is an elite-preserving, scalable, and computationally fast population-alteration model (Ref. 27). This model was chosen because it has been shown to have a faster convergence rate on standard test functions as compared to other evolutionary algorithms and classical optimization algorithms including other real-parameter GAs with the unimodal normal distribution crossover (UNDX) and the simplex crossover (SPX) operators, the correlated self-adaptive evolution strategy, the covariance matrix adaptation evolution strategy (CMA-ES), the differential evolution technique, and the quasi-Newton method (Ref. 27). The original G3 model applied by Kumar et al. (Ref. 28), Kumar and DebRoy (Ref. 20), and Mishra and DebRoy (Refs. 21, 22) for different welding applications has very high selectivity, since at every iteration individuals are created using the best parent and two randomly chosen individuals. The high selec-

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**Table 1 — Terminology Used in Genetic Algorithm**

<table>
<thead>
<tr>
<th>Biological terms</th>
<th>Equivalent welding variables and representation in genetic algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Genes: Units containing hereditary information</td>
<td>In the form of nondimensional variables, $f_1$, $f_2$, $f_3$, and $f_4$, e.g., $f_1 = 1.10$; $f_2 = 1.70$; $f_3 = 1.56$; $f_4 = 1.34$.</td>
</tr>
<tr>
<td>Chromosome/individual: A number of genes folded together</td>
<td>A set of input variable values taken together, i.e., $(1.10, 1.70, 1.56, 1.34)$</td>
</tr>
<tr>
<td>Population: Collection of many chromosomes/individuals</td>
<td>Collection of multiple sets: $(1.10, 1.70, 1.56, 1.34)$, $(1.20, 1.54, 1.65, 1.27)$, ...</td>
</tr>
<tr>
<td>Parents: Chromosomes/individuals participating for creating new individuals (or offsprings)</td>
<td>Parents: e.g., $(1.10, 1.70, 1.56, 1.34)$, $(1.23, 1.65, 1.75, 1.45)$</td>
</tr>
</tbody>
</table>

Equation 3

\[
O(f) = \left( \frac{p'}{p} - 1 \right)^2 + \left( \frac{t'}{t} - 1 \right)^2 + \left( \frac{f'}{f} - 1 \right)^2
\]

where $p'$, $t'$, and $f'$ are the computed penetration, throat, and leg length of the weld bead, respectively, and $p$, $t$, and $f$ are the corresponding target or desired values of these three parameters. The objective function, $O(f)$, depends on four main welding variables, i.e., current, I, voltage, V, welding speed, U, and the wire feed rate, $w_f$.

**Objective function:**

\[
O(f) = O(f_1, f_2, f_3, f_4) = O(I, V, U, w_f)
\]

**Objective function:**

In Equation 3, the reference values, $I_r$, $V_r$, $U_r$, and $w_f$ represent the order of magnitude of the welding variables. Note that
tivity tends to draw the whole population of solutions toward one side of the parameter space. In order to maintain diversity in the population, a modified version of the generalized generation gap (G3) model is used in this work. Here we use three randomly chosen parents to create new individuals in place of best parent and two randomly chosen individuals in the original algorithm.

Results and Discussion

The neural network used here was trained and validated with results from a well-tested three-dimensional numerical heat transfer and fluid flow model. A large database of outputs for different welding conditions was generated based on design of experiments (DOE) (Ref. 29) to capture the correlations between the welding variables and the weld attributes. Separate feed-forward neural networks were developed, one each for predicting penetration, leg length, and throat of a GMA fillet weld in spray mode to achieve high accuracies in the calculation of penetration, leg length, and throat. The weights in the neural network models were calculated using a hybrid optimization scheme involving the conjugate gradient (CG) method and a genetic algorithm (GA). The network was trained using only the training data. The validation and testing data were randomly generated independent of the training data. The performance of the network was tested using the validation and testing datasets. The testing data were used to check the overall performance of the network. The hybrid optimization scheme helped in finding optimal weights through a global search as evidenced by good agreement between all the outputs from the neural networks and the corresponding results from the heat and fluid flow model as shown in Fig. 2.

These results are obtained for welding of A-36 steel plates using argon with 10% CO₂ as shielding gas and solid feed wire of 1.32-mm diameter. The droplet transfer mechanism during welding is assumed to be in spray mode. The workpiece was 450 mm in length, 108 mm in width, and 18 mm in depth. The nominal composition of A-36 steel is maximum 0.29% C, 0.80–1.2% Mn, 0.04% P, 0.05% S, 0.15–0.3% Si, and remaining percentage of Fe. The neural network model provided correct values of penetration, actual throat, and leg length for various combinations of welding variables I, V, U, and w, as shown in Fig. 2. Since GA can provide a population of solutions, the neural network model must be combined with an appropriate GA to tailor weld attributes.

The effectiveness of the model proposed here was tested by finding different sets of welding variables that could provide a specified weld geometry based on scientific principles. The computational task involved three steps. First, a target weld geometry was selected by specifying one set of values of penetration, throat, and leg length. Second, the model was run to obtain multiple combinations of welding variable sets each of which could produce the target weld geometry. Third, and final, the results obtained from the model were adequately verified. These three steps are explained in detail in the next section.

To start the calculation, the specification of a target geometry was necessary. It involved stating realistic combinations of the three weld dimensions, i.e., penetration, throat, and leg length. To test the model, these three weld dimensions from an actual welding experiment were specified as a target geometry. If the model works correctly, the various combinations of welding variables obtained from the model must include a set of welding variables that are fairly close to the set of variables used in the experiment. It should be noted that the ability of the model to produce this solution is only a necessary, but not sufficient, component of the model verification. Since the model produced multiple solutions, other solutions obtained from the model had to be verified by comparing the calculated weld geometry
with the experimentally obtained geometry.

In the next step (i.e., second step), a population of 200 individuals was defined to start the operation of GA. Each individual in the population defined a set of randomly chosen welding variables such as current, voltage, welding speed, and wire feed rate. Choice of an appropriate population size was important. A small population size did not allow adequate representation of the variable space. On the other hand, a very large population size resulted in large computational volume. For example, Fig. 3A depicts the initial values of the individuals, i.e., sets of I, V, U, and w_f of each individual solution with I and V plotted as their product in the form of input arc power. Values of the welding variables I, V, U, and w_f were chosen randomly in the range of 250–400 A, 27–35 V, 3.5–7.0 mm/s, and 150–250 mm/s, respectively. The values of the welding variables in such large ranges helped in maintaining diversity in the solutions. These welding variable sets were then improved iteratively using a combination of GA and the neural network. With the progress in the calculations, the average objective function values decreased with iterations. An individual with a low objective function indicates correct combinations of current, voltage, welding speed, and wire feed rate that can result in the target weld geometry. Figure 3B shows the computed values of the objective functions for all the individuals depicted in Fig. 3A. This figure shows that for many sets of welding variables, the values of the objective function were very close, within less than 1%, to the target geometry.

Figure 4 shows that the objective function decreased rapidly with iterations for the best individual whereas the average value of the objective function of the whole population decreased at a relatively slower pace. This behavior is consistent with the fact that as GA tries to explore the solution space, it produces new sets of welding parameters that have higher values of \( O(f) \). Figure 5A indicates several individuals that for each set of computed welding parameters were then compared with those used to produce the experimental weld. Solution (a) in Table 2 shows values of leg length, penetration, and throat that are very close, within less than 1%, to those used to produce the experimental weld. This table also includes values of other variable sets, i.e., current, voltage, welding speed, and wire feed rate, computed by the model to produce the desired values of leg length, penetration, and throat. Each solution, i.e., a set of current, voltage, welding speed, and wire feed rate was used to calculate weld geometric parameters. The computed geometric parameters were then compared with those produced in the experiment. Table 2 shows that for each set of computed welding con-

Table 2 — The Various Combinations of Welding Parameters, i.e., Arc Current (I), Arc Voltage (V), Welding Speed (U), and Wire Feed Rate (w_f)

<table>
<thead>
<tr>
<th>Individual Solutions</th>
<th>I (Amp)</th>
<th>V (Volt)</th>
<th>U (mm/s)</th>
<th>w_f (mm/s)</th>
<th>Penetration (mm)</th>
<th>Leg Length (mm)</th>
<th>Throat (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>285.1</td>
<td>33.1</td>
<td>4.2</td>
<td>172.4</td>
<td>1.7</td>
<td>10.5</td>
<td>7.2</td>
</tr>
<tr>
<td>(b)</td>
<td>293.3</td>
<td>32.6</td>
<td>4.3</td>
<td>211.0</td>
<td>1.6</td>
<td>10.6</td>
<td>7.1</td>
</tr>
<tr>
<td>(c)</td>
<td>298.3</td>
<td>31.3</td>
<td>4.5</td>
<td>216.2</td>
<td>1.6</td>
<td>10.5</td>
<td>7.2</td>
</tr>
<tr>
<td>(d)</td>
<td>290.8</td>
<td>33.5</td>
<td>4.6</td>
<td>210.0</td>
<td>1.6</td>
<td>10.5</td>
<td>7.2</td>
</tr>
<tr>
<td>(e)</td>
<td>292.2</td>
<td>33.0</td>
<td>4.6</td>
<td>213.0</td>
<td>1.6</td>
<td>10.5</td>
<td>7.1</td>
</tr>
<tr>
<td>(f)</td>
<td>303.3</td>
<td>30.6</td>
<td>4.6</td>
<td>210.5</td>
<td>1.6</td>
<td>10.3</td>
<td>7.3</td>
</tr>
<tr>
<td>(g)</td>
<td>294.1</td>
<td>31.4</td>
<td>4.9</td>
<td>227.0</td>
<td>1.6</td>
<td>10.5</td>
<td>7.2</td>
</tr>
<tr>
<td>(h)</td>
<td>294.7</td>
<td>31.0</td>
<td>5.0</td>
<td>231.0</td>
<td>1.6</td>
<td>10.5</td>
<td>7.2</td>
</tr>
</tbody>
</table>

Fig. 5 — Several fairly diverse welding variable sets could produce low values of the objective function, indicating the existence of alternate paths to obtain the target weld geometry. The plots show the welding variable sets that produced low values of the objective function, \( O(f) \), with iterations. A — individuals after 1000 iterations with \( O(f) \) less than \( 1 \times 10^{-4} \); B — individuals after 3000 iterations with \( O(f) \) less than \( 1 \times 10^{-5} \); and C — individuals after 6000 iterations with \( O(f) \) less than \( 1 \times 10^{-5} \).
and reliable welds. It is hoped that the methodology will serve as basis for formulating, testing, and implementing realistic computational tools for tailoring weld attributes to achieve defect free, structurally sound, and reliable welds.

**Conclusions**

Unlike conventional heat transfer and fluid flow models that can predict weld geometry for a particular set of welding conditions, a new model has been developed that can calculate alternative welding conditions needed to obtain a target weld geometry. The model developed is significantly different from traditional reverse models that provide only one set of welding conditions necessary for obtaining a target weld geometry. In reality, a particular weld geometry can be obtained by using various combinations of welding variables and the new model can calculate these alternative pathways. The model combines a neural network model of heat and fluid flow with a real-number-based genetic algorithm to calculate alternative welding conditions needed to obtain a target weld geometry for GMA fillet welding. The use of a neural network model in place of a heat transfer and fluid flow model significantly increased computational efficiency and provided multiple solutions within one minute in a commonly available computer.

The model was used to determine multiple sets of welding variables, i.e., combinations of welding variables to achieve a desired weld geometry in less than a minute with a commonly available PC. It is hoped that the methodology will serve as basis for formulating, testing, and implementing realistic computational tools for tailoring weld attributes to achieve defect free, structurally sound, and reliable welds.

### Appendix: PCX-Based G3 Genetic Algorithm

The genetic algorithm used in the present study is a parent centric recombination (PCX) operator-based generalized generation gap (G3) model (Refs. 20–22, 27, 28). The steps involved in the calculations are as follows:

1. A population is a collection of many individuals and each individual represents a set of randomly chosen values of the four nondimensionalized welding variables. A parent refers to an individual in the current population. The best parent is the individual that has the best fitness, i.e., gives the minimum value of the objective function, defined by Equation 2 in the main text, in the entire population. Three parents are chosen randomly from the population of solutions.

2. From the three randomly chosen parents, two offsprings or new individuals are generated using a recombination scheme. PCX-based G3 models are known to converge rapidly when three parents and two offsprings are selected (Ref. 27). A recombination scheme is a process for creating new individuals from the parents.

3. Two new parents are randomly chosen from the current population of the individuals.

4. A subpopulation of four individuals that includes the two randomly chosen parents in step 3 and two new offsprings generated in step 2 is formed.

5. The two best solutions, i.e., the solutions having the least values of the objective function, are chosen from the subpopulation of four members created in step 4. These two individuals replace the two parents randomly chosen in step 3.

6. The calculations are repeated from step one again until convergence is achieved.

The above steps, as applied to the present problem, are shown in Fig. 6. Figure 7 illustrates the working of the model to find the window of welding parameters to achieve a target weld geometry. The recombination scheme (step 2) used in the

### Table 3 — The Various Combinations of Welding Parameters, i.e., Arc Current (I), Arc Voltage (V), Welding Speed (U), and Wire Feed Rate (w_f) Obtained Using Neural Network Model to Achieve the Following Target Weld Dimensions: Penetration = 3.7 mm, Leg Length = 12.0 mm, and Throat = 10.0 mm. The Target Weld Geometry Was Obtained Using the Welding Conditions Listed in (a).

<table>
<thead>
<tr>
<th>Individual Solutions</th>
<th>I (Amp)</th>
<th>V (Volt)</th>
<th>U (mm/s)</th>
<th>w_f (mm/s)</th>
<th>Penetration (mm)</th>
<th>Leg Length (mm)</th>
<th>Throat (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>301.6</td>
<td>34.6</td>
<td>3.4</td>
<td>228.6</td>
<td>3.7</td>
<td>12.0</td>
<td>10.0</td>
</tr>
<tr>
<td>(b)</td>
<td>306.2</td>
<td>34.6</td>
<td>3.6</td>
<td>236.6</td>
<td>3.7</td>
<td>12.0</td>
<td>10.0</td>
</tr>
<tr>
<td>(c)</td>
<td>300.3</td>
<td>34.6</td>
<td>3.3</td>
<td>225.9</td>
<td>3.7</td>
<td>12.0</td>
<td>10.0</td>
</tr>
<tr>
<td>(d)</td>
<td>311.0</td>
<td>35.3</td>
<td>4.5</td>
<td>270.8</td>
<td>3.7</td>
<td>12.0</td>
<td>10.0</td>
</tr>
<tr>
<td>(e)</td>
<td>290.8</td>
<td>35.4</td>
<td>4.1</td>
<td>260.2</td>
<td>3.7</td>
<td>12.0</td>
<td>10.0</td>
</tr>
<tr>
<td>(f)</td>
<td>314.1</td>
<td>33.5</td>
<td>3.8</td>
<td>239.0</td>
<td>3.7</td>
<td>11.8</td>
<td>10.0</td>
</tr>
</tbody>
</table>
present model is based on a parent centric recombination (PCX) operator (Refs. 20–22, 27, 28). A brief description of this operator, as applied to the present problem, is presented below:

First three parents, i.e.,

\[
\begin{pmatrix}
 f_1^0, f_2^0, f_3^0, f_4^0 \\
 f_1^1, f_2^1, f_3^1, f_4^1 \\
 f_1^2, f_2^2, f_3^2, f_4^2
\end{pmatrix},
\]

are randomly selected from the current population. Here the superscripts represent the four variables or the welding parameters, while the subscripts denote the parent identification number. The mean vector or centroid, \( \bar{x} \), is next calculated from \( \bar{x} = \frac{1}{3} \left( f_1^0 + f_1^1 + f_1^2, f_2^0 + f_2^1 + f_2^2, f_3^0 + f_3^1 + f_3^2, f_4^0 + f_4^1 + f_4^2 \right) \). of the three chosen parents is computed. To create an offspring, one of the parents, say, \( \bar{x}^{(r)} = \left( f_1^{0(r)}, f_2^{0(r)}, f_3^{0(r)}, f_4^{0(r)} \right) \), is chosen randomly. The direction vector, \( \bar{d}^{(r)} \), is next calculated from the selected parents to the mean vector or centroid. Thereafter, from each of the other two parents, i.e.,

\[
\begin{pmatrix}
 f_1^{1(r)}, f_2^{1(r)}, f_3^{1(r)}, f_4^{1(r)} \\
 f_1^{2(r)}, f_2^{2(r)}, f_3^{2(r)}, f_4^{2(r)}
\end{pmatrix},
\]

perpendicular distances, \( D_i \), to the direction vector, \( \bar{d}^{(r)} \), are computed and their average, \( \bar{D} \), is found. Finally, the offspring, i.e.,

\[
y = \left( f_1^1, f_2^1, f_3^1, f_4^1 \right) \]

is created as follows:

\[
y = \frac{2}{3} \left( f_1^0 - f_1^1 - f_1^2 \right)
\]

where \( h^{(i)} \) are the orthonormal bases that span the subspace perpendicular to \( \bar{d}^{(r)} \), and \( w^1, w^2, w^3, w^4 \) are randomly calculated zero-mean normally distributed variables. The values of the variables that characterize the offspring,

\[
y = \left( f_1^r, f_2^r, f_3^r, f_4^r \right)
\]

are calculated as follows:

\[
f_1^r = f_1^0 + f_1^1 + f_1^2 \quad (A2.a)
\]

\[
f_2^r = f_2^0 + f_2^1 + f_2^2 \quad (A2.b)
\]

\[
f_3^r = f_3^0 + f_3^1 + f_3^2 \quad (A2.c)
\]

\[
f_4^r = f_4^0 + f_4^1 + f_4^2 \quad (A2.d)
\]

where,

\[
f_{11} = w^1 \left( \frac{2}{3} \left( f_1^0 - f_1^1 - f_1^2 \right) \right) \quad (A3.a)
\]

\[
f_{21} = w^2 \left( \frac{2}{3} \left( f_2^0 - f_2^1 - f_2^2 \right) \right) \quad (A3.b)
\]

\[
f_{32} = w^3 \left( \frac{2}{3} \left( f_3^0 - f_3^1 - f_3^2 \right) \right) \quad (A3.c)
\]

\[
f_{42} = w^4 \left( \frac{2}{3} \left( f_4^0 - f_4^1 - f_4^2 \right) \right) \quad (A3.d)
\]
WELDING RESEARCH

\[ f_{\omega} = \frac{w}{\eta} \left( \frac{a_1 + b_1}{2} \right) \left[ 1 - \left( \frac{f_{a1}^0 - f_{a1}^1}{3d} \right)^2 \right] \]

(A3h)

The various unknown variables used in Equations A3.a to A3.h can be represented in simplified form as follows:

\[ d = \frac{\left( f_{a1}^1 - f_{a1}^0 \right)^2 + \left( f_{a2}^0 - f_{a2}^1 \right)^2 + \left( f_{a3}^0 - f_{a3}^1 \right)^2}{3} \]

(A4a)

\[ a_2 = e_1 \times \left( 1 - \left( a_1 \right)^2 \right) \]

(A4b)

\[ b_2 = e_2 \times \left( 1 - \left( b_1 \right)^2 \right) \]

(A4c)

\[ a_1 = \sum_{i=1}^{m} d \times e_1 \]

(A4d)

\[ e_1 = \sqrt{\left( f_{a1}^0 - f_{a1}^1 \right)^2 + \left( f_{a2}^0 - f_{a2}^1 \right)^2 + \left( f_{a3}^0 - f_{a3}^1 \right)^2} \]

(A4e)

\[ b_1 = \sum_{i=1}^{m} d \times e_2 \]

(A4f)

\[ e_2 = \sqrt{\left( f_{a2}^0 - f_{a2}^1 \right)^2 + \left( f_{a3}^0 - f_{a3}^1 \right)^2 + \left( f_{a4}^0 - f_{a4}^1 \right)^2} \]

(A4g)

References